



Date: 31-10-2018  
Time: 01:00-04:00

Dept. No.

Max. : 100 Marks

**Part – A:**

**Answer ALL the questions:**

**(10x2=20)**

1. Define unbiased estimator. Give an example.
2. State any two regularity conditions.
3. State any two properties of UMVUE.
4. When  $X_1, X_2, X_3, \dots, X_n$  is random samples from Poisson distribution with parameter  $\lambda$ , suggest a sufficient statistic.
5. Explain the Method of Moment estimation.
6. State the least Square estimator of  $\beta_0$ , in the model  $Y = \beta_0 + \beta_1 X + \varepsilon$ .
7. What is a prior distribution? Give an example.
8. State a possible prior distribution for  $\lambda$
9. State the 95% confidence interval for  $\mu$ , when a random sample of size 'n' is drawn from  $N(\mu, 1)$ .
10. With  $\alpha_1 < \alpha_2$ , which of the two confidence intervals  $(1-\alpha_1)100\%$  and  $(1-\alpha_2)100\%$  is wider?

**Part – B**

**Answer any FIVE questions:**

**(5x8=40)**

11. Derive an unbiased estimator of 'p' in Bernoulli distribution  $B(1, p)$ , based on a random sample of size 'n'.
12. Describe efficient estimator with a suitable example.
13. Define completeness of an estimator and verify whether  $\bar{x}$  is a complete estimator in case of a random sample of size 'n' from  $N(\theta, 1), \theta \in R$ .
14. Describe the Method of minimum Chi – square estimation.
15. State any four properties of MLE.
16. Describe Loss function and Risk function with suitable example.

17. Describe posterior distribution? Give an example of a prior and posterior distribution.
18. Derive the 95% confidence interval for  $\theta$ , when a random sample of size 'n' drawn from  $N(\mu, \sigma^2)$ ,  $\theta \in R$ , where  $\sigma^2$  is unknown.

**Part - C**

**Answer any TWO Questions:** **(2x20=40)**

19. a) State and prove Cramer – Rao inequality. **(12)**
- b) Show that the family of Poisson distribution  $\{P(\lambda), \lambda > 0\}$  is complete. **(8)**
20. a) State and prove Lehmann – Scheffe theorem. **(10)**
- b) State and prove Rao – Blcakwell theorem. **(10)**
21. a). Explain the concept of estimation by the method of modified minimum chi-square. **(10)**
- b) For a random sample of  $X_1, X_2, X_3, \dots, \dots, X_n$  from  $N(\mu, \sigma^2)$ ,  $\sigma > 0, \mu \in \mathcal{R}$ , obtain the method of moments estimator of  $\mu$  and  $\sigma^2$ . **(10)**
22. a) Let  $X_1, X_2, X_3, \dots, \dots, X_n$  be a random sample from  $\{P(\lambda), \lambda > 0\}$ , and let Gamma  $(\alpha, \beta)$  be prior distribution for  $\lambda$ . Obtain the Bayesian estimator for  $\lambda$ . **(12)**
- b) If  $X_1, X_2, X_3, \dots, \dots, X_n$  and  $Y_1, Y_2, Y_3, \dots, \dots, Y_m$  are random sample of size 'n' and 'm' respectively from  $N(\mu_1, \sigma^2)$  and  $N(\mu_2, \sigma^2)$ . Derive the  $(1 - \alpha)100\%$  confidence interval for the differences in means  $\mu_1 - \mu_2$ . **(8)**

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